

Math Academy Application Test

Summer 2022

Try to solve as many problems as you can, and for each of them show your reasoning. If you are stuck on a problem, write down what you have figured out.

Problem 1. Draw eight points in such a way that:

1. no three points lie on a common line; and
2. no five points are the vertices of a convex pentagon.

Problem 2. A high school mathematics class has three pairs of twins. Not one of these twins is friends with their twin sibling. One of the twins, called X, notices that each of the other five twins is friends with a different number of twins in the class. How many twins in the class is X friends with? How many twins in the class is X's twin friends with?

Problem 3. Draw a regular hexagon and all of its nine diagonals, as in Figure 1. Choose two colours, say red or blue. Colour each diagonal with either red or blue in such a way that there are only three pairs of diagonals that cross each other and have the same colour. Can you colour each diagonal with either red or blue in such a way that there are only two pairs of diagonals that cross each other and have the same colour?

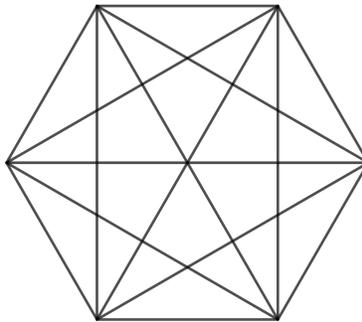


Figure 1: a regular hexagon and all of its nine diagonals.

Problem 4. There are eight kids who want to play a game of pick-up hockey. To create two teams of four players, the two best players will take turns picking teammates. They toss a coin to determine who gets to pick first. Is the following reasoning correct? Explain why or why not.

The number of different teams that could be formed in this way is $2 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 1440$. This is because there are 2 possibilities for who wins the coin toss, then the winner of the coin toss has 6 possible choices for their first pick, then the loser of the coin toss has 5 possible choices for their first pick, then the winner of the coin toss has 4 possible choices for their second pick, then the loser of the coin toss has 3 possible choices for their second pick, then the winner of the coin toss has 2 possible choices for their third pick, and finally the loser of the coin toss has 1 possible choice for their third pick.