Background Information

What is a polygon?

A polygon is a region of the plane whose border is a path made up of straight line segments which only touch at endpoints (called vertices).

A regular polygon has all its sides the same length and the angles are all the same. These are the most commonly encountered types by students and tend to have standard names (triangle, square, pentagon, hexagon, heptagon, octagon, etc). You can do various calculations on regular polygons, many involving angles and side lengths.

Polyhedra are regions of 3D space with boundaries made entirely of polygons (the faces) which only touch by sharing an entire edge. Polyhedra should only be convex in nature and should be regular meaning that all faces are the same regular polygon and the vertices are all identical. In this case we can do many interesting and useful calculations on the angles, vertices and shape of the polygon sides.

Polytopes are made up of n – 1 polytopes (called faces) and exist in n-dimensional space. For example a 6-polytope would have 5-polytope faces and would exists in 5D space.

Visualizing polytopes is tricky in that it requires a visualization of the image of the n-space below the n-polytope you are investigating. For example when investigating a 4D polytope it is useful to concentrate on its 3-d boundary: this is a bunch of 3-d polyhedra which are glued together along their 2-d faces.

Recall that we sometimes describe 3-d polyhedra by describing their 2-d surface. The most obvious case of this is its net, a collection of planar polygons which folds to give the surface of the polyhedron. The net of a polytope is a collection of polyhedra glued along their faces: the result is the 3-d boundary of the 4-d polytope.

It is therefore important for students to be able to grasp this net for 3D shapes in order to more fully understand n-dimensional shapes when studying regular polytopes.

Activity -- Making 3D shapes with nets

What’s needed:
- thick paper
- scissors
- pens/pencils/markers
- rulers
- glue
- geometric nets (provided)
Classroom Adventures in Mathematics – Geometry (26 Jan 2013)

How to do:
- Use the geometric nets provided and copy them on plain or coloured paper
- Have students cut and fold the paper and create geometric shapes, compare them to various other polyhedra and identify the underlying polygon shapes that make up their substructure
- Discuss the variations of angles and the “wiggle room” angle

Variations
- depending on the grade level of the students you may wish to make as many simple or complex shapes as you wish, including basic shapes (triangle, square and other 2D shapes)
- for advanced students they should be taking measurements of angles and seeing how small variations in these angles affect the overall symmetry of the shapes
- for younger students discussions on the symmetry of the shapes can be done

Curriculum Links

These lessons can be applied to any areas of the Elementary school curriculum guidelines under the “Geometry and Spatial Sense” headings. The variations allow them to be useful at various grade levels and to hit on a variety of topics contain within the various areas.

At the high school level, these lessons can be useful for:

- Grade 9 Analytic Geometry and Measurement and Geometry sessions
- Grade 10 Analytic Geometry and can be used in discussions of Trigonometry of angles and triangles (many investigations on various faces can done using Trigonometric equations)
- Grade 11 Functions can use various functions to do angle, side and other calculations on the shapes
- Grade 12 Calculus and Vectors can make use of the larger more complex shapes to study vector geometry and solve angles and areas using various algebraic and trigonometric equations and solutions.

Resources

- PowerPoint Slides from Presentation
- Illuminations Website on Making Shapes
  o [http://illuminations.nctm.org/](http://illuminations.nctm.org/)
- Playing with Polygons
  o [http://www.mathsisfun.com/geometry/polygons-interactive.html](http://www.mathsisfun.com/geometry/polygons-interactive.html)
- Animated Polyhedrons
  o [http://www.mathsisfun.com/geometry/polyhedron-models.html](http://www.mathsisfun.com/geometry/polyhedron-models.html)